$$\pm\frac{x\sqrt{[1+(dx/dy)^2+(dx/dz)^2]}}{y\sqrt{[1+(dy/dx)^2+(dy/dz)^2]}}=a=\pm\frac{xdx}{ydy}\ldots(1).$$

$$\pm\frac{x\sqrt{[1+(dx/dy)^2+(dx/dz)^2]}}{z\sqrt{[1+(dz/dx)^2+(dz/dy)^2]}}=b=\pm\frac{xdx}{zdz}\ldots(2).$$

$$\pm\frac{y\sqrt{[1+(dy/dx)^2+(dy/dz)^2]}}{z\sqrt{[1+(dz/dx)^2+(dz/dy)^2]}}=c=\pm\frac{ydy}{zdz}\ldots(3).$$

$\therefore aydy=xdx,\ bzdz=xdx,\ czdz=ydy,\ aydy+bzdz+czdz=2xdx+ydy.$
$\therefore 2x^2=(b+c)z^2+(a-1)y^2+D.$
$\therefore x^2=Ay^2+Bz^2+D$ are the surfaces satisfying the conditions.

The equation could be put in the form $x^2=ay^2-acz^2+D=0$, where $ac=-b$, as is the case from (1), (2), and (3).

323. Proposed by W. J. GREENSTREET, Marling School, Stroud, England.

S, S' are the foci of two co-vertical parabolas A and B, the axes of which are at right angles. Draw the circle K on SS' as diameter. K is cut in D and E by a straight line parallel to the axis of A such that S' lies midway between it and that axis. Show that the lines $S'D$, $S'E$ are parallel to the two tangents to A which are normals to B.

Solution by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philadelphia, Pa.

The problem as stated is not true.

Let $y^2=4ax$, $x^2=4by$, be the parabolas A, B; $y=mx+a/m$=tangent to A; $y-y'=m(x-x')$ the normal to B.

Since $x'^2=4by'$, $m=-2b/x'$ or $x'=-2b/m$, $y'=x'^2/4b=b/m^2$.

$\therefore y=mx+2b+b/m^2$=normal to B.

But $y=mx+a/m$ and $y=mx+2b+b/m^2$ are the same line.

$\therefore a/m=2b+b/m^2$.

$\therefore m=\dfrac{1}{4b}[a\pm\sqrt{(a^2-8b^2)}]$, which is true for $a \gtreqless 2\sqrt{2}\,b$.

$x^2-ax+y^2-by=0$ is equation to K, $y=2b$=line parallel to axis of A.

$\therefore x^2-ax+2b^2=0$ or $x=\frac{1}{2}[a\pm\sqrt{(a^2-8b^2)}]$.

$\therefore y=\dfrac{2bx}{a\pm\sqrt{(a^2-8b^2)}}+b$ are the equations to $S'D$, $S'E$.

$\therefore$ If m were twice as great, $S'D$, $S'E$ would be perpendicular to the two tangents to A which are normals to B.

The lines through S' parallel to the two tangents are given by the equation

$$y=\frac{1}{4b}[a\pm\sqrt{(a^2-8b^2)}]x+b.$$

This line intersects K in the points

$$\left[-\frac{6ab^2 \pm 2b^2\sqrt{(a^2-8b^2)}}{a^2+4b^2 \pm a\sqrt{(a^2-8b^2)}},\ -\frac{a^2b-12b^3 \pm ab\sqrt{(a^2-8b^2)}}{a^2+4b^2 \pm a\sqrt{(a^2-8b^2)}}\right],$$

the plus and minus signs to be used together.

$$y=\frac{8abx}{a^2+4b^2}+12b^3-a^2b+\frac{48a^2b^3}{a^2+4b^2+a\sqrt{(a^2-8b^2)}}$$

is the line through these points. The tangents to A parallel to $S'D$, $S'E$ are

$$y=\frac{2bx}{a\pm\sqrt{(a^2-8b^2)}}+\frac{a[a\pm\sqrt{(a^2-8b^2)}]}{2b}$$

This line meets $x^2=4by$ in

$$x_1=\frac{4b^2\pm\sqrt{\{16b^4+2a[a\pm\sqrt{(a^2-8b^2)}]^3\}}}{a\pm\sqrt{(a^2-8b^2)}}=r.$$

The tangent at this point makes an angle with the axis of abscissas whose tangent is $x_1/2b$. As this does not equal $-1/m$ the problem, as stated, is not true.

CALCULUS.

252. Proposed by J. H. MEYER, S. J., Augusta, Ga.

Supposing the arc of a semi-circle to be stretched out into a straight line, and an indefinite number of perpendiculars erected on it, each equal to the versed sine of the corresponding arc; what would be the length of the curve traced out by the tops of the perpendiculars?

Solution by CHAS. O. GUNTHER, Acting Professor of Mathematics, Stevens Institute of Technology, Hoboken, N. J.

Assuming a as the radius of the circle, the equation of the curve is $y=\text{vers}\ x/a$, and the required length of the curve is given by the expression

$$s=2\int_0^{\pi a/2}\left(1+\frac{\sin^2 x/a}{a^2}\right)dx=2\int_0^{\pi a/2}\sqrt{(a^2+1-\cos^2 x/a)}\frac{dx}{a}.$$

Let $\cos x/a=\sin\theta$; then

$$s=2\sqrt{(a^2+1)}\int_0^{\frac{1}{2}\pi}\sqrt{1-\left(\frac{1}{\sqrt{(a^2+1)}}\right)^2\sin^2\theta}\,d\theta=2\sqrt{(a^2+1)}E\left(\frac{1}{\sqrt{(a^2+1)}},\ \tfrac{1}{2}\pi\right).$$

Also solved by G. B. M. Zerr.